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Rapid Note

Universal scaling properties of extreme type-II superconductors in magnetic fields

T. Schneider, J. Hofer, M. Willemin, J.M. Singer[a], and H. Keller

Physikinstitut, Universität Zürich, Winterthurerstr. 190, 8057 Zürich, Switzerland

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Abstract. Anisotropic Ginzburg-Landau superconductors of extreme type-II are considered in an approximation where magnetic field fluctuations are neglected. A formulation of the scaling properties is presented for the singular part of the free energy density in the presence of a magnetic field. From the existence of a magnetization, a diamagnetic susceptibility and superconductivity we determine the limiting behavior of the scaling function in the vicinity of the zero field transition temperature, where critical fluctuations dominate. Our predictions for the temperature and field dependence of magnetization, magnetic torque and melting line *etc.*, uncover the universal critical properties and provide an extension of hitherto used mean-field treatments. The results are consistent with experimental data.

PACS. 74.25.-q General properties; correlations between physical properties in normal superconducting states – 05.70.Jk Critical point phenomena

There is considerable evidence that the zero-field transition in high T_c superconductors is a critical point belonging to the universality class of the three dimensional XY model [1–8]. The absence of mean-field like behavior can be understood by noting that thermal fluctuations are much larger in these materials than in conventional superconductors, since their free energy density multiplied by the correlation volume is comparable to the thermal energy over a significant temperature range around T_c [9]. However, when vector potential fluctuations are taken into account, one concludes that this transition might be different [10]. For extreme type-II superconductors, such as high-T_c materials, the coupling to vector potential fluctuations is weak. Consequently, there is a range of temperatures close, but not asymptotically close, to T_c, in which vector potential fluctuations can be neglected. Therefore the critical behavior is that of the XY model, which also describes the superfluid transition in ^{4}He.

Thus, in the presence of a small applied magnetic field H, the singular part of the free energy density f_s is expected to exhibit a scaling behavior similar to that obtained in the Gaussian approximation. Starting from a Ginzburg-Landau action with an effective mass anisotropy (M_x, M_y, M_z) and an applied field $\mathbf{H} = (0, H_y, H_z) = H(0, \sin(\delta), \cos(\delta))$, the singular part of the free energy density adopts the scaling form

$$f_s = \frac{k_B T Q_1^\pm}{\xi_x^\pm \xi_y^\pm \xi_z^\pm} G^\pm(z), \quad G^\pm(0) = 1, \tag{1}$$

where $G^\pm(z)$ is a universal scaling function with scaling variable

$$\begin{aligned} z &= \frac{(\xi_x^\pm)^2}{\Phi_0} \sqrt{\frac{M_x}{M_z} H_y^2 + \frac{M_x}{M_y} H_z^2} \\ &= \frac{(\xi_x^\pm)^2 H}{\Phi_0} \sqrt{\frac{M_x}{M_z} \sin^2(\delta) + \frac{M_x}{M_y} \cos^2(\delta)}, \end{aligned} \tag{2}$$

where $\pm = sign(t)$ and $t = T/T_c - 1$. In zero field (*i.e.* $z = 0$) one recovers for $G^\pm(0) = 1$ the scaling expression for the 3D XY universality class [11,12]. $\xi_i^\pm$ is the correlation length along direction i, which diverges as $\xi_i^\pm = \xi_{i,0}^\pm |t|^{-\nu}$, where $\nu \approx 2/3$, and the $Q_1^\pm$ are universal numbers [11,12]. We note that in the isotropic case ($M_x = M_y = M_z$) this scaling form was confirmed by perturbation theory [13].

In principle, renormalization group calculations should allow an unambiguous calculation of the scaling function G. In the low field regime, however, the Landau energy levels of Cooper pairs are closely spaced and must all be taken into account. Under this circumstances the difficulties which result from the expansion of the order parameter in terms of the Landau levels have impeded the calculation of the scaling function.

[a] e-mail: jms@physik.unizh.ch

In this letter, we describe an alternative approach. The functional form of $G(z)$ for relevant regimes of the scaling variable z are derived from the existence of the magnetization, the diamagnetic susceptibility and a superconducting phase in the appropriate domain of z. Using the resulting limiting behavior of $G(z)$, the scaling properties of any observable is then obtained *via* the singular part of the free energy density. Here we concentrate on magnetization, susceptibility, melting line and magnetic torque.

For $H_y = 0$ we obtain from the free energy density, equation (1), for the fluctuation contribution to the diamagnetic magnetization, $m_z = -df_s/dH_z$, the scaling relation

$$m_z = -\frac{Q_1^\pm k_B T}{\Phi_0}\frac{\xi_x^\pm}{\xi_y^\pm \xi_z^\pm}\sqrt{\frac{M_x}{M_y}}\frac{dG^\pm}{dz},$$
$$z = \sqrt{\frac{M_x}{M_y}}\frac{(\xi_{x,0}^\pm)^2|t|^{-2\nu}H_z}{\Phi_0}\cdot \quad (3)$$

Its existence at $t = 0$ and finite H_z (*i.e.* $z \to \infty$) leads to the universal relation

$$\frac{m_z}{H_z^{1/2}T_c}\frac{\xi_{y,0}^\pm \xi_{z,0}^\pm}{(\xi_{x,0}^\pm)^2}\left(\frac{M_y}{M_x}\right)^{3/4} = -\frac{Q_1^\pm k_B C_\infty^\pm}{\Phi_0^{3/2}},$$
$$C_\infty^\pm = \lim_{z\to\infty} z^{-1/2}\frac{dG^\pm}{dz}, \quad (4)$$

and determines $G(z)$ in this limit. Thus, the combination of measurable properties entering the left hand should be independent of the material under investigation. This relation appears to be well confirmed for $YBa_2Cu_3O_{7-\delta}$ [5]. For small fields and $t > 0$ (*i.e.* $z \to 0$) the existence of the susceptibility, $\chi = m_i/H_i$ $(i = x, y, z)$, implies together with equation (1)

$$\chi_z = -\frac{Q_1^+ k_B T C_0^+ (\xi_{x,0}^+)^3}{\Phi_0^2 \xi_{y,0}^+ \xi_{z,0}^+}\frac{M_x}{M_y}|t|^{-\nu},$$
$$C_0^+ = \lim_{z\to 0}\frac{dG^+(z)/dz}{z}\cdot \quad (5)$$

Another quantity of interest is $\partial m_i/\partial \ln H_i = H_i \partial m/\partial H_i$ for $t < 0$ and $H \to 0$. Taking the scaling form equation (1) for the singular part of the free energy density we find

$$\lim_{H_z\to 0} H_z\frac{\partial m_z}{\partial H_z} = -\frac{Q_1^- \Phi_0}{16\pi^3\lambda_x^2}\sqrt{\frac{M_x}{M_y}}C_{2,0}^-,$$
$$C_{2,0}^- = \lim_{z\to 0} z\frac{d^2G^-(z)}{dz^2}, \quad (6)$$

by invoking the universal relation between T_c and the critical amplitudes of the transverse correlation length,

$$k_B T_c = (\Phi_0^2 \xi_{i,0}^T)/(16\pi^3\lambda_{i,0}^2),$$

which is based on the existence of a superconducting phase for $t < 0$ and $H \to 0$. Here we used $\xi_i^T = \xi_{i,0}^T|t|^{-\nu}$, as well as $\lambda_i = \lambda_{i,0}|t|^{-\nu/2}$ for the penetration depth, and $\xi_x^- = \sqrt{\xi_y^T\xi_z^T}$, $\xi_y^- = \sqrt{\xi_x^T\xi_z^T}$, $\xi_z^- = \sqrt{\xi_x^T\xi_y^T}$ [2]. Thus, this limit indeed requires the existence of superconductivity. The resulting logarithmic dependence of m_z on H_z is also obtained from the London model [14], which assumes that the vortex cores do not overlap; this condition is satisfied for $z = H\xi_x^2/\Phi_0 \ll 1$ and consistent with $z \to 0$ for $t < 0$.

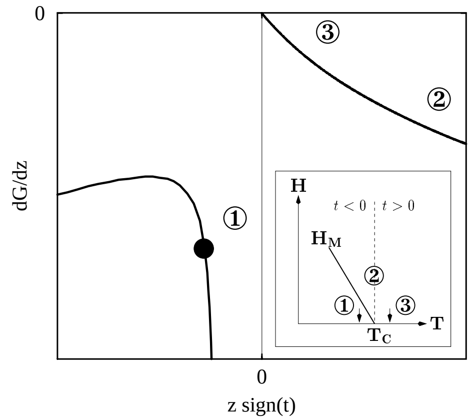

Fig. 1. Qualitative behavior of the universal scaling function in terms of dG/dz *vs.* $zsign(t)$. the black dot marks the possible location of the melting transition. The arrows indicate the limits 1, 2 and 3 considered here. The inset shows the corresponding limits in the (H, T)-plane, where H_M is the melting line.

We have shown that the limits

$$\lim_{z\to 0} dG^-(z)/dz = C_{2,0}^- \ln(z),$$
$$\lim_{z\to\infty} dG^\pm(z)/dz = C_\infty^\pm z^{1/2},$$
$$\lim_{z\to 0} dG^+(z)/dz = C_0^+ z, \quad (7)$$

of the universal scaling function can be derived from the existence of superconductivity, the magnetization and the susceptibility. The resulting qualitative behavior of dG/dz *versus* $zsign(t)$ is shown in Figure 1, where the arrows 1, 2 and 3 indicate the limits listed in equation (7). The inset shows the corresponding behavior in the (H, T)-plane. As phase transition lines are concerned, on application of a magnetic field at temperatures below T_c the mean-field approximation predicts that in type-II superconductors the transition occurs in two stages: if the magnetic field is reduced one first encounters at $H_{c2}(T)$ a transition from the normal state to a mixed phase, and then, at a lower critical field $H_{c1}(T)$, a transition to the Meissner state. The two critical lines meet at the zero-field transition point T_c. Experimentally, however, the thermodynamic properties of high-T_c cuprates show no evidence for a sharp anomaly at some $H_{c2}(T)$, but there is strong evidence for a first order phase transition line [15–17], which ends at the zero field transition point T_c and describes the vortex lattice melting.

Thus, by approaching the critical endpoint T_c along the melting line the transition becomes gradually second order [15]. Given this fact, the scaling function $G(z)$ should have a singularity at some value z_M of the scaling variable, so that according to equation (2) the temperature and angular dependence of the melting line near the multicritical point T_c follows from

$$z_M = \frac{(\xi_{x,0}^-)^2 H_M(T)}{\Phi_0}\left(\frac{M_x}{M_z}\sin^2(\delta)+\frac{M_x}{M_y}\cos^2(\delta)\right)^{1/2} \times\left(\frac{T_c(H=0)-T}{T_c(H=0)}\right)^{-2\nu}. \quad (8)$$

This behavior appears to be fully consistent with the experimental data for the melting [15–17] and irreversibility lines of $YBa_2Cu_3O_{7-\delta}$ [18].

Next we consider the magnetic torque, where the pronounced effective mass anisotropy of the high-T_c materials enters in an essential way. Using the scaling expression for the singular part of the free energy density, equation (1), we obtain for the fluctuation contribution to the torque, $\mathcal{T}=\mathbf{m}\times\mathbf{H}$, the expression

$$\mathcal{T}_x = \frac{Q_1^\pm k_B T(\xi_x^\pm)^3}{\Phi_0^2\xi_y^\pm\xi_z^\pm}\frac{1}{z}\frac{dG^\pm(z)}{dz}\frac{M_x}{M_y}\left(1-\frac{M_y}{M_z}\right) \times H^2\sin(\delta)\cos(\delta). \quad (9)$$

Clearly, there is no torque in an isotropic superconductor (*i.e.* $M_x=M_y=M_z$). We note, that (i) the field-, angular- and temperature-dependence of the torque can be measured very accurately [19,20], (ii) various mean-field models have been proposed and used to extract the effective mass anisotropy and penetration depth in high-T_c materials [14,19,21,22]. Thus, the strong evidence for critical fluctuations, as cited above, calls for a more rigorous treatment. It can be readily obtained by invoking the scaling approach in the limits given by equation (7). Indeed, from equations (7, 9) we obtain for $t<0$, $H\to 0$ (*i.e.* limit 1 in Fig. 1) the expression

$$\mathcal{T}_x = \frac{Q_1^-\Phi_0 C_{2,0}^- H}{16\pi^3\lambda_x^2}\sqrt{\frac{M_x}{M_y}}\left(1-\frac{M_y}{M_z}\right) \times\frac{\cos(\delta)\sin(\delta)}{\sqrt{\cos^2(\delta)+\frac{M_y}{M_z}\sin^2(\delta)}}\ln(z), \quad (10)$$

while for $t=0$, $H\neq 0$ (*i.e.* limit 2 in Fig. 1)

$$\mathcal{T}_x = \frac{Q_1^+ C_\infty^+ k_B T_c(\xi_{x,0}^+)^2}{\xi_{y,0}^+\xi_{z,0}^+\Phi^{3/2}}\left(\frac{M_x}{M_y}\right)^{3/4}\left(1-\frac{M_y}{M_z}\right) \times H^{3/2}\frac{\cos(\delta)\sin(\delta)}{\left(\cos^2(\delta)+\frac{M_y}{M_z}\sin^2(\delta)\right)^{1/4}}, \quad (11)$$

and for $t>0$, $H\to 0$ (*i.e.* limit 3 in Fig. 1)

$$\mathcal{T}_x = \frac{Q_1^+ C_0^+ k_B T(\xi_{x,0}^+)^3|t|^{-\nu}}{\Phi_0^2\xi_{y,0}^+\xi_{z,0}^+}\frac{M_x}{M_y}\left(1-\frac{M_y}{M_z}\right) \times H^2\sin(\delta)\cos(\delta). \quad (12)$$

It is important to emphasize that the scaling approach is not restricted to the so far considered gauge

$$\mathbf{H}=H(0,\sin(\delta),\cos(\delta)).$$

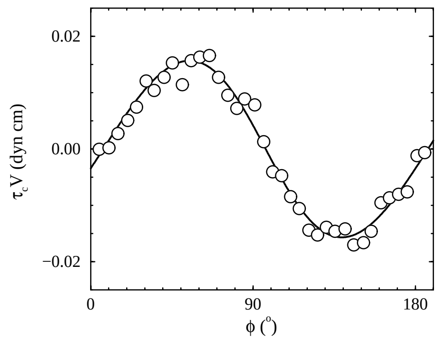

Fig. 2. Magnetic torque $\mathcal{T}_c V$ of untwinned $YBa_2Cu_3O_{7-\delta}$ with $T_c = 91.7$ K and $V = 0.89\times 10^{-3}$ cm^3, upon rotating around the c-axis at $H = 8$ kOe and $T = 90$ K. The solid line corresponds to equation (10) with $x=c$, $y=b$ and $z=a$, for the critical parameters cited in the text, the open circular symbols are experimental data taken from [23].

Indeed, $\mathbf{H}=H(\sin(\delta),0,\cos(\delta))$ leads to $\mathcal{T}_y$ and $\mathbf{H}=H(\cos(\phi),\sin(\phi),0)$ to $\mathcal{T}_z$, and the scaling variables adopt the forms

$$z\Phi_0 = (\xi_y^\pm)^2\left((M_y/M_z)H_x^2+(M_y/M_x)H_z^2\right)^{1/2} \quad \text{and}$$
$$z\Phi_0 = (\xi_z^\pm)^2\left((M_z/M_y)H_x^2+(M_z/M_x)H_y^2\right)^{1/2},$$

respectively.

We are now prepared to analyze experimental magnetic torque data, taken in the appropriate limits, to uncover the potential of the scaling approach and to illustrate the estimation of fundamental critical properties. As the limit 1 is concerned, a rather complete study has been performed with an untwinned $YBa_2Cu_3O_{7-\delta}$ single crystal in reference [23] upon rotating the field around the a, b and c axes. Using the angular dependence of their torque data for the fields rotated around the b and a axes we obtain by means of our scaling approach (*i.e.* Eq. (10)) the estimates: $\xi_{b,0}^-\approx 14.7$ Å, $\sqrt{M_c/M_b}\approx 8.95$, $\sqrt{M_b/M_a}\approx 0.84$, $\xi_{a,0}^-=\sqrt{M_b/M_a}\xi_{b,0}^-\approx 12.4$ Å and $\xi_{c,0}^-=\sqrt{M_b/M_c}\xi_{b,0}^-=1.64$ Å. The critical amplitudes of the penetration depth follow then *via* the universal relation $k_BT_c=(\Phi_0^2\xi_{i,0}^T)/(16\pi^3\lambda_{i,0}^2)$, rewritten in the form $k_BT_c=(\Phi_0^2\xi_{a,0}^-\sqrt{M_b/M_c})/(16\pi^3\lambda_{b,0}^2)$, and we obtain $\lambda_{a,0}\approx 1153$ Å, $\lambda_{b,0}\approx 968$ Å and $\lambda_{c,0}\approx 8705$ Å. The quality of these critical parameters relies on the applicability of equation (10), requiring that $z\ll 1$. This condition is reasonably well satisfied, $0.02\le z\le 0.15$, for $H=8$ kOe, $T=90$ K and $T_c=91.7$ K, where the experiment has been performed [23]. Given this set of critical amplitudes and the torque at angle δ, normalized with respect to the sample volume, the universal constant $Q_1^-C_{20}^-$ in equation (10) is then readily estimated: $Q_1^-C_{20}^-\approx 0.68$. On this basis, the torque for a field rotated around the c-axis can be calculated without any adjustable parameter. Indeed, for $\mathbf{H}=H(\cos(\phi),\sin(\phi),0)$ the scaling expression corresponds to equation (10) with $x=c$, $y=b$ and $z=a$. Using the parameters cited above, we plotted in Figure 2 the angular dependence of $\mathcal{T}_cV$. For comparison

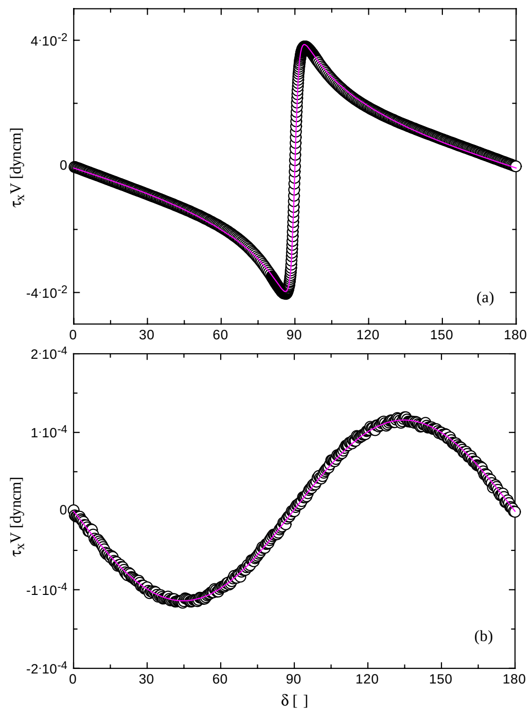

Fig. 3. Magnetic torque $\mathcal{T}_x(\delta)$ for a $HgBa_2CuO_{4+\delta}$ with $T_c =$ 95.6 K and $V = 1.26\times10^{-6}$ cm^3 at $H = 14$ kOe, upon rotating the field from the c-axis to an unknown direction in the ab-plane. The upper figure (a) shows data for $T = 90.9$ K and the lower one for $T = 104.5$ K. The solid lines correspond to nonlinear least square fits to equation (10) in panel (a) and equation (12) in panel (b).

we included the experimental data from [23]. Because there $0.007 \leq z \leq 0.0083$, the condition for the applicability of equation (10) is well satisfied and the remarkable agreement with the experimental data points to a consistent description of the experimental torque data of [23] in terms of our estimates for the fundamental critical amplitudes.

Finally, to explore the applicability of the three dimensional scaling approach to materials with more pronounced effective mass anisotropy, and to provide experimental evidence for the limit 3 as well as for the universality of $Q_1^- C_{2,0}^-$, we proceed with an analysis of the angular dependence of reversible torque data of a $HgBa_2CuO_{4+\delta}$ single crystal with $T_c = 95.6$ K, taken at $H = 14$ kOe and various temperatures, upon rotating the field from the c-axis to an unknown direction in the ab-plane. Experimental details are described elsewhere [20, 24]. Figure 3a shows data for $T = 90.9$ K, where equation (10) is expected to apply with $M_a \approx M_b \equiv M_\parallel$. The solid line shows the best fit to equation (10), yielding $\sqrt{M_c/M_\parallel} \approx 28.8$ and $\xi_{\parallel,0}^- \approx 27$ Å, $0.01 \leq z \leq 0.27$. Thus, the condition for the applicability of equation (10) is reasonably well satisfied. Invoking then again the universal relation $k_B T_c = (\Phi_0^2 \xi_{\parallel,0}^- \sqrt{M_\parallel/M_c})/(16\pi^3 \lambda_{\parallel,0}^2)$, we obtain $\lambda_{\parallel,0} \approx 784$ Å and $Q_1^- C_{2,0}^- \approx 0.71$. Recalling the previous $YBa_2Cu_3O_{7-\delta}$-result $Q_1^- C_{2,0}^- \approx 0.68$, the experimental verification of the universality of $Q_1^- C_{2,0}^-$ appears to be established.

In the limit 3, where equation (12) applies, the torque is expected to exhibit the simple $\sin(\delta)\cos(\delta)$ behavior. As shown in Figure 3b, this feature is experimentally well confirmed in $HgBa_2CuO_{4+\delta}$ at $T = 104.5$ K and $H = 14$ kOe, where $0.0006 \leq z \leq 0.017$ with $\xi_{\parallel,0}^+ \approx 10.5$ Å. Here we used the universal relation $\xi_{\parallel,0}^+ = \xi_{\parallel,0}^- (\mathcal{R}^+/\mathcal{R}^-)(\mathcal{A}^-/\mathcal{A}^+)^{1/3}$ with $\mathcal{R}^+ \approx 0.36$, $\mathcal{R}^- \approx 0.96$ and $\mathcal{A}^+ \approx \mathcal{A}^-$ [2,11,12]. The fit to equation (12) yields for the universal constant the estimate $Q_1^+ C_0^+ \approx 0.17$. In the limit 2 equation (11) applies as long as $z \gg 1$, which requires that $(\xi_\parallel^+)^2 \gg (M_\perp \Phi_0)/(M_\parallel H)$. Unfortunately, we are not aware of torque data which satisfy this condition.

In conclusion, we presented a formulation of the scaling properties of the singular part of the free energy density in the presence of a magnetic field for anisotropic extreme type-II superconductors. Our results for the temperature and field dependence of magnetization, magnetic torque, melting line, *etc.* uncover the universal critical properties and provide a basis to handle and extract the effective mass anisotropy as long as the continuum version of the Ginzburg-Landau description applies. Moreover, these results provide an extension of hitherto used mean-field treatments of the magnetic torque and are consistent with experimental data.

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References

1. A. Kozlowski, *et al.*, Physica C **184**, 113 (1991).
2. T. Schneider, H. Keller, Int. J. Mod. Phys. B **8**, 487 (1994).
3. N. Overend, *et al.*, Phys. Rev. Lett. **72**, 3238 (1994).
4. S. Kamal, *et al.*, Phys. Rev. Lett. **73**, 1845 (1994).
5. M.A. Hubbard, *et al.*, Physica C **259**, 309 (1996).
6. M.A. Howson, *et al.*, Phys. Rev. B **51**, 11984 (1995).
7. Y. Jaccard, *et al.*, Europhys. Lett. **34**, 281 (1996).
8. M. Roulin, *et al.*, Physica C **260**, 237 (1996).
9. D.S. Fisher, *et al.*, Phys. Rev. B **43**, 130 (1991).
10. I. Herbut, *et al.*, Phys. Rev. Lett. **76**, 4599 (1996).
11. P.C. Hohenberg, *et al.*, Phys. Rev. B **13**, 2986 (1976).
12. V. Privman, *et al.*, in *Phase Transitions and Critical Phenomena*, edited by C. Domb, J.L. Lebowitz, Vol. 14 (Academic Press, 1991).
13. I.D. Lawrie, Phys. Rev. Lett. **79**, 131 (1997).
14. V.G. Kogan, Phys. Rev. B **38**, 7049 (1988).
15. R. Liang, *et al.*, Phys. Rev. Lett. **76**, 835 (1996).
16. A. Schilling, *et al.*, Phys. Rev. Lett. **78**, 4833 (1997).
17. M. Roulin, *et al.*, to appear in Physica C (1998).
18. J.R. Cooper, *et al.*, Phys. Rev. Lett. **79**, 1730 (1997).
19. D.E. Farrell, *et al.*, Phys. Rev. Lett. **61**, 2805 (1988).
20. D. Zech, *et al.*, Phys. Rev. B **53**, R6026 (1996).
21. N. Bulaevski, *et al.*, Phys. Rev. B **47**, 8056 (1991).
22. A. Buzdin, *et al.*, Physica C **220**, 74 (1994).
23. T. Ishida, *et al.*, Phys. Rev. B **56**, 11897 (1997).
24. J. Hofer, *et al.*, Physica C **297**, 103 (1998).